**2** **2.1** В силу потенциальн. поле

$$W_1 = \varphi_1 q' = \frac{kqq'}{r_1} \qquad W_2 = \varphi_2 q' = \frac{kqq'}{r_2}$$

$$A_{12} = W_1 - W_2 = \frac{qq'}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

В случае б) $A = 0$, т.к $r_1 = r_2$.

**2.2**


$$\varphi_0 = 2\sum_{\alpha=1}^{\infty} \frac{k(-1)^\alpha q}{\alpha a} = \frac{2kq}{a}\sum_{\alpha=1}^{\infty}\frac{(-1)^\alpha}{\alpha} =$$

$$= -\frac{q}{2\pi\varepsilon_0 a}\ln 2.$$

**2.3** а) $\varphi_0^{(1)} = \frac{kq}{a/2} \qquad \varphi_0^{(2)} = -\frac{kq}{a/2} \Rightarrow \varphi_0 = \varphi_0^{(1)} + \varphi_0^{(2)} = 0$

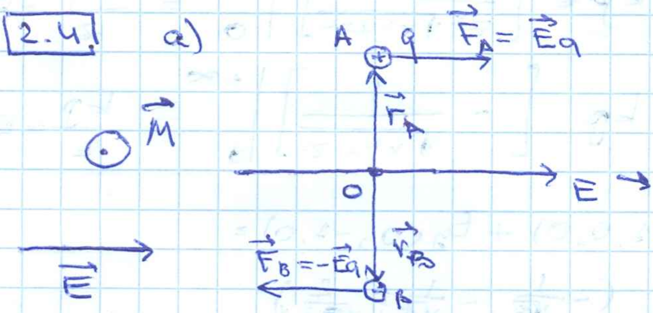
$$\varphi_\infty = 0$$

тогда $A = q'(\varphi_0 - \varphi_\infty) = 0$.

б) $\varphi_0^{(1)} = \varphi_0^{(2)} = \frac{kq}{a/2} \qquad \varphi_0 = \frac{kq}{a} \qquad \varphi_\infty = 0$

$$A = q'(\varphi_0 - \varphi_\infty) = +\frac{kqq'}{a}.$$

**2.4** а)




$$W = q(\varphi_A - \varphi_B) = $$
$$= q\int_A^B \vec E\, d\vec r = -q\vec E\vec l = 0.$$

$x$: $F_A - F_B = Eq - Eq = 0.$

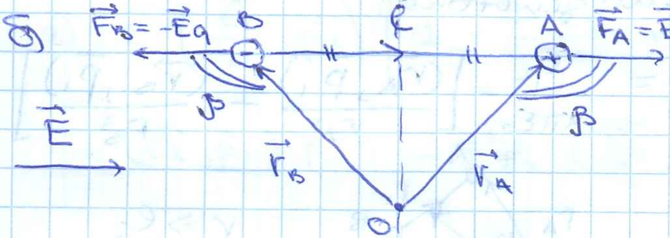
$$\vec F_\Sigma = 0$$

$$\vec{M} = [\vec{r}_A \times \vec{F}_A] + [\underset{-\vec{r}_A}{\vec{r}_B} \times \underset{-\vec{F}_A}{\vec{F}_B}] = [\vec{r}_A \times \vec{E}q] + [\vec{r}_A \times \vec{E}q] =$$

$$= 2[\vec{r}_A \times \vec{E}q] = [\vec{p}_e \times \vec{E}]$$ (направлено на нас)

$$M = p_e E \sin\frac{\pi}{2} = p_e E \qquad (2\vec{r}_A = \vec{\ell})$$

а)



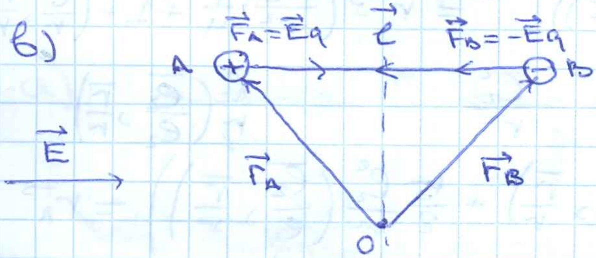

$x: F_A - F_B = Eq - Eq = 0$

$\vec{F}_\Sigma = 0$

$$\vec{M} = [\vec{r}_A \times \vec{F}_A] + [\vec{r}_B \times \underset{-\vec{F}_A}{\vec{F}_B}] = [\underset{\vec{\ell}}{\vec{r}_A - \vec{r}_B}, \vec{F}_A] = 0$$

$$W = q(\varphi_A - \varphi_B) = q\int_A^B \vec{E}\,d\vec{r} = -q\underset{\cos\alpha = \cos 0 = 1}{\vec{E}\vec{\ell}} = -Ep_e$$

б)




$x: F_A - F_B = Eq - Eq = 0$

$\vec{F}_\Sigma = 0$

$$\vec{M} = [\vec{r}_A \times \vec{F}_A] + [\vec{r}_B \times \underset{-\vec{F}_A}{\vec{F}_B}] = [\underset{\vec{\ell}}{\vec{r}_A - \vec{r}_B}, \vec{F}_A] = 0$$

$$W = q(\varphi_A - \varphi_B) = q\int_A^B \vec{E}\,d\vec{r} = -q\underset{\cos\alpha = \cos\pi = -1}{\vec{E}\vec{\ell}} = Ep_e$$

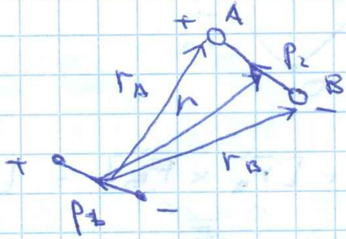
$\boxed{2.5}$ Потенциал диполя в т. M (1-ый способ)

$$\varphi(M) = \frac{(\vec r, \vec p_e)}{4\pi\varepsilon_0 \cdot r^3}$$

Тогда энергия взаимодействия:

$$W_2 = q_2(\varphi_A - \varphi_B) = \frac{q_2}{4\pi\varepsilon_0}\left[\frac{(\vec r_A, \vec p_1)}{r_A^3} - \frac{(\vec r_B, \vec p_1)}{r_B^3}\right]$$

$$\vec r_A = \vec r + \frac{\vec \ell_2}{2}$$




$r \gg \ell$

$$\vec r_B = \vec r + \frac{\vec \ell_2}{2}$$

$$r_A^2 = \left(\frac{\bar\ell_2}{2} + \bar r\right)^2 = \frac{\ell_2^2}{4} + r^2 + (\bar\ell_2, \bar r)$$

$$r_A^{-3} = \left(\frac{\ell_2^2}{4} + r^2 + (\bar\ell_2, \bar r)\right)^{-3/2} = r^{-3}\left(\frac{\ell_2^2}{r^2}\cdot\frac{1}{4} + 1 + \left(\frac{\bar\ell}{r}, \frac{\bar r}{r}\right)\right)^{-3/2}$$

раскладываем до $\frac{\ell_2^2}{r^2}$ $\qquad \frac{\ell_2}{r}\left(\frac{\bar\ell_2}{\ell_2}, \frac{\bar r}{r}\right)$

$$= r^{-3}\left(1 - \frac{3}{8}\frac{\ell_2^2}{r^2} - \frac{3}{2}\frac{\ell_2}{r}\left(\frac{\bar\ell_2}{\ell_2}, \frac{\bar r}{r}\right) + \frac{15}{8}\frac{\ell_2^2}{r^2}\left(\frac{\bar\ell_2}{\ell_2}, \frac{\bar r}{r}\right)\right) = r_A^{-3}$$

$$r^{-3}\left(1 - \frac{3}{8}\frac{\ell_2^2}{r^2} + \frac{3}{2}\frac{\ell_2}{r}\left(\frac{\bar\ell_2}{\ell_2}, \frac{\bar r}{r}\right) + \frac{15}{8}\frac{\ell_2^2}{r^2}\left(\frac{\bar\ell_2}{\ell_2}, \frac{\bar r}{r}\right)\right) = r_B^{-3}$$

$$(\bar r_A, \bar p_1) = (\bar r, \bar p_1) + \left(\frac{\bar\ell_2}{2}, \bar p_1\right)$$

$$(\bar r_B, \bar p_1) = (\bar r, \bar p_1) - \left(\frac{\bar\ell_2}{2}, \bar p_1\right)$$

1) $(\bar r, \bar p_1)\left[-3\frac{l_2}{r}\left(\frac{\bar l_2}{l_2}, \frac{\bar r}{r}\right)\right] r^{-3} = -3\frac{l_2}{r}\,\frac{(\bar r, \bar p_1)\left(\frac{\bar l_2}{l_2}, \frac{\bar r}{r}\right)}{r^3} =$

$$= \frac{-3(\bar r, \bar p_1)(\bar l_2, \bar r)}{r^5}$$

2) $2\left(\frac{\bar l_2}{2}, \bar p_1\right)\left[1 - \frac{3}{8}\frac{l_2^2}{r^2} + \frac{15}{8}\frac{l_2^2}{r^2}\left(\frac{\bar l_2}{l_2}, \frac{\bar r}{r}\right)\right] r^{-3} =$

$$= (\bar l_2, \bar p_1) r^{-3} - \frac{3}{8}\frac{l_2^3}{r^3}\left(\frac{\bar l_2}{l_2}, \bar p_1\right) r^{-2} + \frac{15}{8}\frac{l_2^3}{r^3}\left(\frac{\bar l_2}{l_2}, \frac{\bar r}{r}\right)\left(\frac{\bar l_2}{l_2}, \bar p_1\right) r^{-2}$$

$$\approx (\bar l_2, \bar p_1) r^{-3}$$

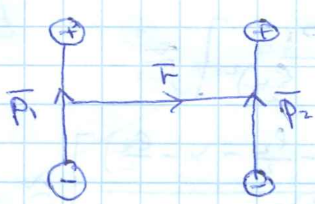
$$W_2 = \frac{q_2}{4\pi\varepsilon_0}\,\frac{(\bar l_2, \bar p_1)}{r^3} + \frac{q_2}{4\pi\varepsilon_0}\,\frac{-3(\bar r, \bar p_1)(\bar r, \bar l_2)}{r^5} =$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec p_1 \cdot \vec p_2}{r^3} - 3\frac{(\vec r, \vec p_1)(\vec r, \vec p_2)}{r^5}\right]$$

a)

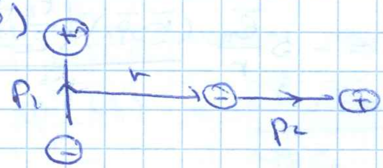

$$W_2 = -\frac{p_1 p_2}{2\pi\varepsilon_0 r^3} \qquad r \gg l_1;\ r \gg l_2$$

б)




$$W_2 = \frac{1}{4\pi\varepsilon_0}\frac{p_1 p_2}{r^3}$$

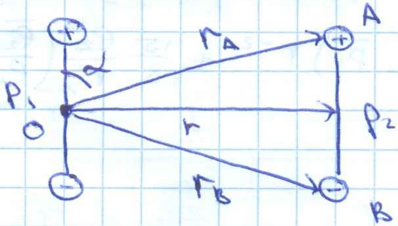
в)




$W_2 = 0$

2-ой способ.

б) $r_0 = \sqrt{r^2 + \frac{\ell_2^2}{4}} \approx r$




$$W_2 = q_2(\varphi_A - \varphi_B) =$$

$$= \frac{q_2}{4\pi\varepsilon_0}\left(\frac{\bar{r}_A \bar{p}_1}{r_A^3} - \frac{\bar{r}_B \bar{p}_1}{r_B^3}\right) =$$

$$= [r_A = r_B = r_0] \overset{1}{\underset{2}{=}}$$

$$\overset{1}{=} \frac{q_2}{4\pi\varepsilon_0}\left[\frac{r_0 p_1 \cos\alpha}{r_0^3} - \frac{r_0 p_1 \cos(\pi-\alpha)}{r_0^3}\right] =$$

$$= \frac{q_2}{4\pi\varepsilon_0}\left[\frac{2 r_0 p_1 \cos\alpha}{r_0^3}\right] = \frac{p_1 p_2}{4\pi\varepsilon_0 r_0^3} \approx \frac{p_1 p_2}{4\pi\varepsilon_0 r^3}$$

$$\overset{2}{=} \frac{q_2}{4\pi\varepsilon_0 r_0^3}\,\bar{p}_1 \bar{\ell}_2 = \frac{p_1 p_2}{4\pi\varepsilon_0 r_0^3} \approx \frac{p_1 p_2}{4\pi\varepsilon_0 r^3}$$

а)

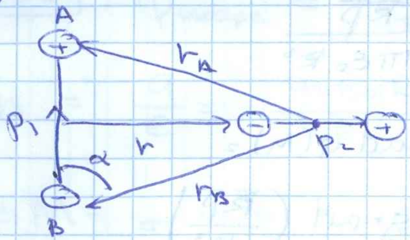

$r = x$

$$W_2 = \frac{q_2}{4\pi\varepsilon_0}\left[-\frac{p_1}{(x-\frac{\ell_2}{2})^2} + \frac{p_1}{(x+\frac{\ell_2}{2})^2}\right] \approx$$

$$= \frac{q_2 p_1}{4\pi\varepsilon_0}\left[\frac{(x+\frac{\ell_2}{2}+x-\frac{\ell_2}{2})(x-\frac{\ell_2}{2}-x-\frac{\ell_2}{2})}{(x^2-\frac{\ell_2^2}{4})^2}\right] \approx$$

$$\approx \frac{q_2 p_1}{4\pi\varepsilon_0}\,\frac{2x\cdot(-\ell_2)}{x^4} \approx -\frac{p_1 p_2}{2\pi\varepsilon_0 x^3}$$

в)



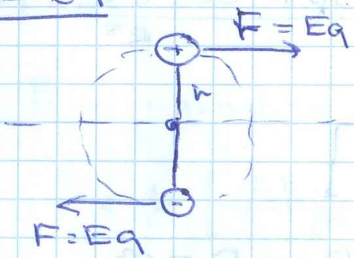
$$W_1 = q_1(\varphi_A - \varphi_B) =$$

$$= \frac{q_1}{4\pi\varepsilon_0}\left(\frac{\bar{r}_A\bar{p}_2}{r_A^3} - \frac{\bar{r}_B\bar{p}_2}{r_B^3}\right) =$$

$$= [r_A = r_B = r_0] =$$

$$= \frac{q_1}{4\pi\varepsilon_0 \cdot r_0^3}\,\bar{p}_2(\bar{r}_A - \bar{r}_B) = \frac{\bar{p}_1\bar{p}_2}{4\pi\varepsilon_0 r_0^3} = 0.$$

$\boxed{2.6}$



$$J = mr^2 + mr^2 = 2mr^2, \quad r = \frac{\ell}{2}$$

$$J\varepsilon = 2Fr$$

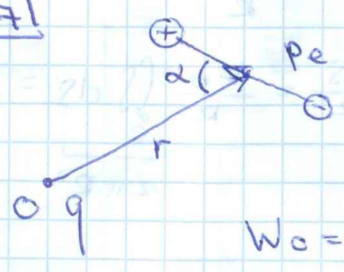
$$\varepsilon = \frac{\cancel{2}Eqr}{\cancel{2}mr^2} = \frac{Eq\frac{\ell}{2}}{m\frac{\ell^2}{4}} = \frac{2Ep_e}{m\ell^2} = \beta$$

из задачи 2.4 имеем: (ЗСЭ)

$$0 = \frac{J\omega^2}{2} - Ep_e \quad ; \quad p_e = q\ell = 2qr$$

$$\omega = \sqrt{\frac{2Ep_e}{J}} = \frac{2}{\ell}\sqrt{\frac{p_eE}{m}}$$

$\boxed{2.7}$



$$W = \varphi(0)q = q\,\frac{(-\bar{r}\,\bar{p}_e)}{4\pi\varepsilon_0 r^3} =$$

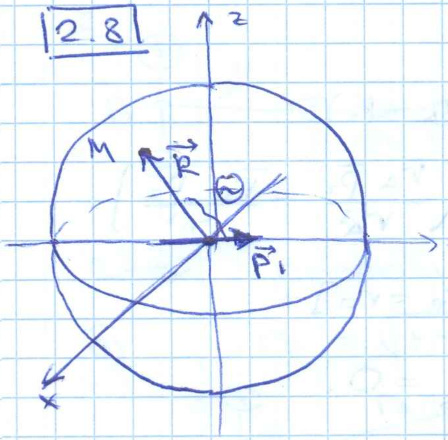
$$\alpha \in [0, \pi] \qquad = \frac{-qrp_e\cos\alpha}{4\pi\varepsilon_0 r^3}$$

$$W_0 = 0 = \frac{-qrp_e\cos\alpha}{4\pi\varepsilon_0 r^3} \Rightarrow \cos\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$$

$$W' = \frac{qrp_e}{4\pi\varepsilon_0 r^3}(+\sin\alpha)$$

$$W_{min} \sim \alpha = 0 \qquad W_{max} \sim \alpha = \pi.$$

2.8




$$\varphi(M) = \frac{\overline{R}\,\overline{p}}{4\pi\varepsilon_0 R^3}$$

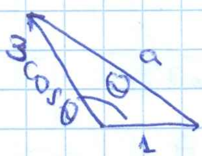
$$\overline{E} = -\operatorname{grad}\varphi(M) =$$

$$= -k \operatorname{grad}\left(\frac{\overline{R}\,\overline{p}}{R^3}\right) =$$

$$= k\left[\frac{3(\overline{p}\,\overline{R})\,\overline{R}}{R^5} - \frac{\overline{p}}{R^3}\right] =$$

$$= k\left[\frac{3pR\cos\Theta\,\overline{R} - R^2\,\overline{p}}{R^5}\right] = k\left[\frac{3pR^2\cos\Theta\,\overline{e}_1 - pR^2\overline{e}_2}{R^5}\right]$$

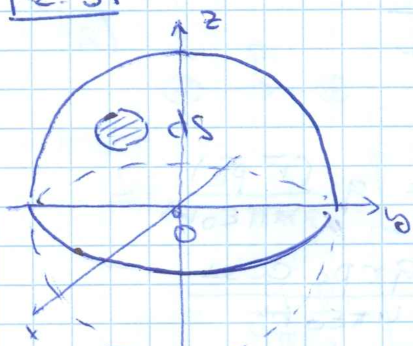
$$= \frac{p}{4\pi\varepsilon_0}\,\frac{1}{R^3}\left[3\cos\Theta\,\overline{e}_1 - \overline{e}_2\right]$$




$$1 + 9\cos^2\Theta - 6\cos\Theta\cos\Theta = a^2$$

$$E = \frac{p}{4\pi\varepsilon_0 R^3}\sqrt{1+3\cos^2\Theta}$$

2.9




$$dq = \sigma\, dS$$

$$d\varphi = \frac{k\,dq}{R} = \frac{k\sigma\, dS}{R}$$

$$\varphi = \iint_S k\sigma\frac{dS}{R} = \frac{k\sigma}{R}\underbrace{\iint_S dS}_{2\pi R^2} = \frac{R\sigma}{2\varepsilon_0}$$

$$E = -\operatorname{grad}\varphi$$

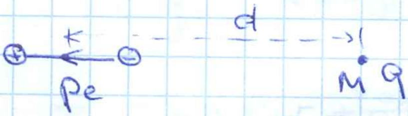
решение далее

**2.10** $\varphi_{\text{шара}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

$$n = \frac{q}{e} = \frac{4\pi\varepsilon_0 R}{e} = 2\cdot 10^{10}$$

$$\Delta M = m\cdot n = 2\cdot 10^{-20}\ \text{кг}.$$

**2.11**




$$\varphi(M) = \frac{\bar r\,\bar p}{4\pi\varepsilon_0 d^3}$$

$$\bar E = -\operatorname{grad}\varphi = k\left[\frac{3(\bar p\bar r)\bar r}{d^5} - \frac{\bar p}{d^3}\right] =$$

$$= k\left[-\frac{3pd\,\bar r}{d^5} - \frac{\bar p}{d^3}\right]$$

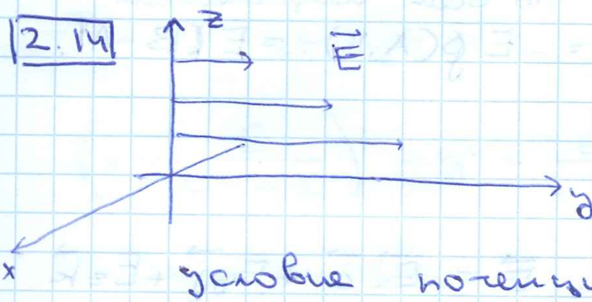
$$E = k\left[-\frac{3pd^2}{d^5} + \frac{p}{d^3}\right] = -\frac{k\,2p}{d^3} = \frac{-p}{2\pi\varepsilon_0 d^3}$$

$$|F| = |Eq| = \frac{qp}{2\pi\varepsilon_0 d^3}$$

$|F| = \frac{qp}{2\pi\varepsilon_0 d^3}$ . в случае ⊕—⊖ •q притяжение

⊖—⊕ •q отталкиван.

**2.14**




$$\vec E = (0, F(z), 0)$$

$$F(z) = \alpha z + \beta,\ \alpha \neq 0$$

условие потенциальност поле:

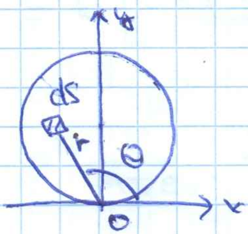
$$\operatorname{rot}\vec E = 0$$

$$\operatorname{rot}\vec{E} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & F(z) & 0 \end{vmatrix} = \bar{i}\left[0 - F'_z(z)\right] + \vec{j}\,0 + \vec{k}\,0$$

$$= -F'_z(z)\,\vec{i} \qquad F'_z = (\alpha z + \beta)' = \alpha \neq 0$$

$\Rightarrow$ поле непотенциально $\Rightarrow$ не может сущ.

$\boxed{2.15}$




$$r = [0; 2R\sin\Theta], \quad \Theta \in [0, \pi]$$

$$dq = \sigma\, dS \qquad d\varphi = \frac{k\sigma\, dS}{r}$$

$$\varphi = \frac{\sigma}{4\pi\varepsilon_0} \iint_S \frac{dS}{r} = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{\pi} d\Theta \int_0^{2R\sin\Theta} \frac{1}{r}\, r\, dr =$$

$$= \frac{\sigma}{4\pi\varepsilon_0} \int_0^{\pi} 2R\sin\Theta\, d\Theta = \frac{2R\sigma}{4\pi\varepsilon_0} \int_0^{\pi} \sin\Theta\, d\Theta =$$

$$= \frac{R\sigma}{2\pi\varepsilon_0} \left[-\cos\Theta\right]\Big|_0^{\pi} = \frac{R\sigma}{\pi\varepsilon_0}$$

$\boxed{2.12}$ Если предположить, что поле $\vec{E}$ однородное по всем направлениям

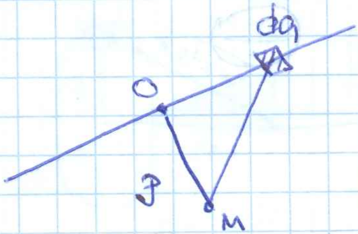
то $\varphi_A - \varphi_B = \int_A^B E\, dr = E\,\rho(A,B) = E\,|B - A|.$

тогда $E_x = \dfrac{\varphi_4 - \varphi_1}{a}$

$E_y = \dfrac{\varphi_4 - \varphi_2}{a}$, $\quad \vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$

$E_z = \dfrac{\varphi_4 - \varphi_3}{a}$

**2.13**




$$dq = \lambda\, dx$$

$$d\varphi = \frac{k\,dq}{r} = \frac{k\lambda\, dx}{\sqrt{x^2+\rho^2}}$$

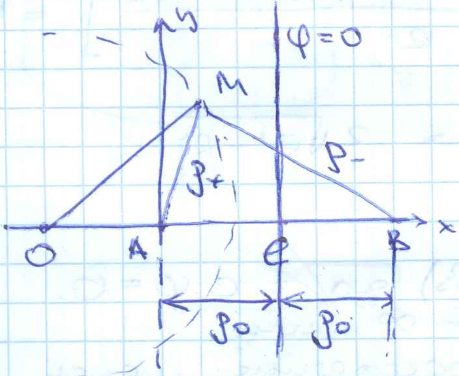
$$\varphi_+ = 2\int_{\rho}^{\rho_0} k\lambda \frac{dx}{\sqrt{x^2+\rho^2}} = 2k\lambda\left[\ln\left(\sqrt{x^2+\rho^2}+x\right)\right]_{\rho}^{\infty} =$$

$$= 2k\lambda \ln\frac{\rho_0}{\rho_+}$$

Плоскость между проводник. имеет потенциал 0.

$$\varphi_- = -2k\lambda \ln\frac{\rho_0}{\rho_-}$$

$\rho_0$ — расстояние от плоскости до проводников




$$\varphi(M) = \varphi_+ + \varphi_- = 2k\lambda \ln\frac{\rho_-}{\rho_+}$$

$$\rho_+ = \sqrt{x^2+y^2}$$

$$\rho_- = \sqrt{(x-l)^2+y^2}$$

$$\frac{\rho_-}{\rho_+} = \sqrt{\frac{(x-l)^2+y^2}{x^2+y^2}} = const = c$$

$$x^2 - 2xl + l^2 + y^2 = c^2x^2 + c^2y^2$$

$$x^2+y^2 + 2\frac{l}{c^2-1}x = \frac{l^2}{c^2-1}$$

$$\left(x+\frac{l}{c^2-1}\right)^2 + y^2 = \frac{l^2}{c^2-1} + \frac{l^2}{(c^2-1)^2} = \frac{c^2l^2}{(c^2-1)^2} = R^2$$

— Окружность с центром $\left(-\frac{l}{c^2-1}, 0\right)$ и радиусом $R = \left|\frac{cl}{c^2-1}\right| \Rightarrow$ цилиндр.

$$\begin{cases} \ell + \dfrac{2\ell}{c^2-1} = \ell\,\dfrac{c^2+1}{c^2-1} = 2a \\ \ell\,\dfrac{c}{c^2-1} = R \qquad (2) \end{cases} \qquad \div$$

$$\frac{c^2+1}{c} = \frac{2a}{R}$$

$$c^2 - 2\frac{a}{R}c + 1 = 0$$

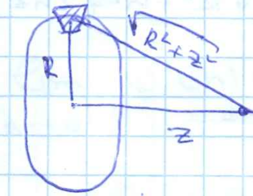
$$c = \frac{a}{R} + \sqrt{\frac{a^2}{R^2} - 1} = \frac{a + \sqrt{a^2-R^2}}{R} \Rightarrow (2)$$

$$\ell\,\frac{a+\sqrt{a^2-R^2}}{R}\left[\frac{a^2 + 2a\sqrt{a^2-R^2} + a^2 - R^2 - R^2}{R^2}\right]^{-1} = R$$

$$\ell = \frac{2a^2 + 2a\sqrt{a^2-R^2} - 2R^2}{a + \sqrt{a^2-R^2}} = 2\sqrt{a^2-R^2}$$

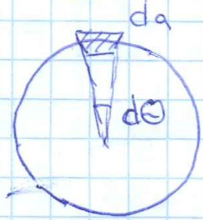
* $\rho_0$ — расстояние от A(B) до т. с $\varphi = 0$.
можно положить $\rho_0 = \infty$ или
заметить, что плоскость посередине между
проводниками и паралл. им имеет $\varphi = 0$.

$\boxed{2.9}$ 1) найдём потенциал кольца в точке на оси $\perp$ плоскости кольца



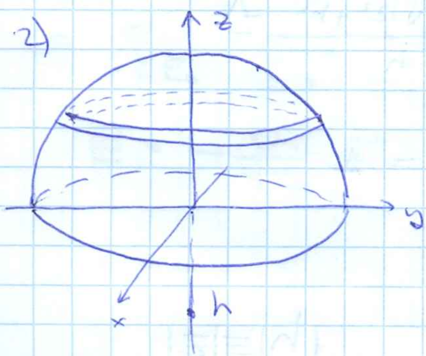

$$dq = \frac{Q}{2\pi}\, d\Theta$$

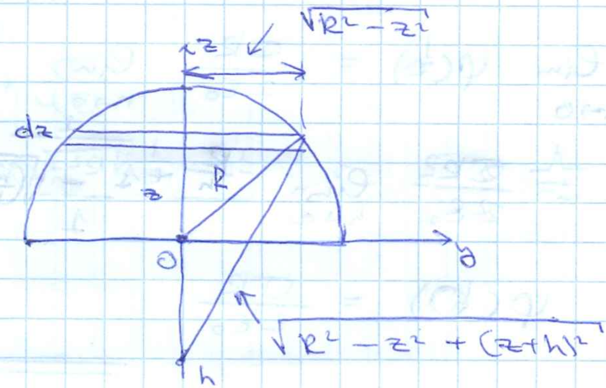
$$d\varphi = \frac{k\,dq}{\sqrt{R^2+z^2}} = \frac{k\frac{Q}{2\pi}\,d\Theta}{\sqrt{R^2+z^2}}$$

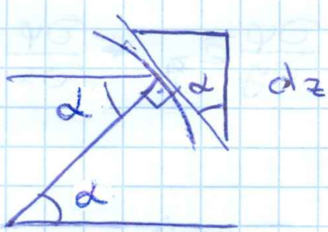



$$\varphi = \int_0^{2\pi} d\varphi = \frac{kQ}{2\pi}\cdot\frac{1}{\sqrt{R^2+z^2}}\int_0^{2\pi} d\Theta = \frac{kQ}{\sqrt{R^2+z^2}}$$

2)




$$\cos\alpha = \frac{\sqrt{R^2-z^2}}{R}$$

$$Q \equiv dq = \sigma\, dS\ ; \quad dS = 2\pi\sqrt{R^2-z^2}\, dz\, \frac{1}{\cos\alpha} =$$

$$= 2\pi\sqrt{R^2-z^2}\,\frac{R}{\sqrt{R^2-z^2}}\, dz = 2\pi R\, dz$$

$$d\varphi = \frac{k\,dq}{r} = 2\pi\sigma R k \cdot \frac{dz}{\sqrt{R^2-z^2+(z+h)^2}}$$

$$\varphi(h) = \frac{\sigma R}{2\varepsilon_0} \int_0^R \frac{dz}{\sqrt{R^2 - z^2 + (z+h)^2}} = \frac{\sigma R}{2\varepsilon_0} \int_0^R \frac{dz}{\sqrt{R^2 + h^2 + 2hz}} =$$

$$= \frac{\sigma R}{2\varepsilon_0} \cdot \frac{1}{+2h} \int_0^R \frac{d(R^2 + h^2 + 2hz)}{\sqrt{R^2+h^2+2hz}} =$$

$$= \frac{\sigma R}{2\varepsilon_0} \frac{1}{2h} \left[ 2\sqrt{R^2+h^2+2hz} \right] \Big|_0^R =$$

$$= \frac{\sigma R}{2\varepsilon_0} \cdot \frac{1}{h} \left[ R + h - \sqrt{R^2+h^2} \right]$$

$$\lim_{h\to 0} \varphi(h) = \frac{\sigma R}{2\varepsilon_0} \lim_{h\to 0} \frac{R+h-\sqrt{R^2+h^2}}{h} \overset{\Lambda}{=}$$

$$\overset{\Lambda}{=} \frac{\sigma R}{2\varepsilon_0} \lim_{h \to 0} \frac{1 - \frac{1}{2}\frac{1}{\sqrt{R^2+h^2}} \cdot 2h}{1} = 1$$

$$\varphi(0) = \frac{\sigma R}{2\varepsilon_0}$$

$h \equiv z$

$$\vec{E} = -\operatorname{grad}\varphi = -\frac{\partial\varphi}{\partial z}\vec{e}_z - \frac{\partial \varphi}{\partial y}\vec{e}_y - \frac{\partial\varphi}{\partial x}\vec{e}_x ;$$

$$\frac{\partial\varphi}{\partial y} = 0 \qquad \frac{\partial \varphi}{\partial x} = 0$$

$$\frac{\partial\varphi}{\partial z} = \frac{\partial}{\partial z}\left( \frac{\sigma R}{2\varepsilon_0} \frac{R+z-\sqrt{R^2+z^2}}{z} \right) =$$

$$= \frac{\sigma R}{2\varepsilon_0} \cdot \frac{\left(1 - \frac{2h}{\sqrt{R^2+h^2} \cdot 2}\right)h - \left(R+h-\sqrt{R^2+h^2}\right)}{h^2} =$$

$$= \frac{\sigma R}{2\varepsilon_0} \cdot \frac{1}{h^2}\left[ \cancel{h} - \frac{h^2}{\sqrt{R^2+h^2}} - R - \cancel{h} + \sqrt{R^2+h^2} \right] =$$

$$= \frac{\sigma R}{2\varepsilon_0} \; \frac{1}{h^2} \left[ -R + \frac{R^2+h^2-h^2}{\sqrt{R^2+h^2}} \right]$$

$$\lim_{h\to 0} \varphi'(h) = \frac{\sigma R}{2\varepsilon_0} \lim_{h\to 0} \left( \frac{-R}{h^2} + \frac{R^2}{\sqrt{R^2+h^2}\cdot h^2} \right) = \left\{ t = \frac{h}{R} \right\}$$

$$= \frac{\sigma R}{2\varepsilon_0} \lim_{t\to 0} \left( \frac{-R + \frac{R^2}{R\sqrt{1-t^2}}}{R^2 t^2} \right) =$$

$$= \frac{\sigma R}{2\varepsilon_0} \lim_{t\to 0} \left( \frac{-R + R\left[1 + t^2 \cdot \frac{1}{2}\right]}{R^2 t^2} \right) =$$

$$= \frac{\sigma}{2\varepsilon_0} \lim_{t\to 0} \frac{+\frac{1}{2}t^2}{t^2} = + \frac{\sigma}{4\varepsilon_0}$$

$$\vec{E} = -\frac{\sigma}{4\varepsilon_0} \vec{e_z}$$